# Simultaneous Heat and Water Model of a Freezing Snow-Residue-Soil System I. Theory and Development

G. N. Flerchinger, K. E. Saxton

ASSOC. MEMBER ASAE  MEMBER ASAE

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## ABSTRACT

Frozen soil is a major cause of runoff and erosion on many watersheds. Tillage and crop residue management greatly affect soil freezing but their effects have been nearly impossible to predict. A detailed, physically-based model is presented which integrates detailed representations for the interrelated heat, water and solute transfer through snow, crop residue and soil. Measured or estimated hourly weather data are used to predict soil freezing depths, evaporation and profiles of temperature, moisture, ice and solutes.

## INTRODUCTION

Frozen soil plays a significant role in the hydrology of many watersheds. Pore blockage by ice greatly decreases the permeability of soil and causes large runoff rates from otherwise mild rainfall or snowmelt events. Extreme erosion rates result when this runoff occurs on super-saturated unprotected soil where the surface may be thawed, but deeper soil is still frozen.

The occurrence, depth and permeability of frozen soil is dependent on the interrelated processes of heat and mass transfer at the soil surface and within the soil profile. In an agricultural setting, management practices can signficantly affect heat and mass transfer by altering surface cover and soil properties. Consequently, defining new methods for runoff and erosion control on agricultural watersheds requires predicting the effects of crop residues and soil properties on heat and mass transfer in the soil-atmosphere micro-climate.

Accurately predicting freezing and thawing of agricultural soils has been nearly impossible because of tillage and residue management effects. Many methods have been developed for predicting soil freezing (Aldrich, 1956; Molnau and Bissell, 1983; Cary et al., 1978; Benoit and Mostaghimi, 1985; Harlan, 1973; Jame and Norum, 1980; and others), but none fully addressed the effects of tillage, residue and snow on the interrelated processes of heat and water transfer. Predictive capabilities for tillage and residue effects on soil frost need to be developed before management options can adequately be evaluated.

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The authors are: G. N. FLERCHINGER, Research Hydrualic Engineer, USDA-Agricultural Research Service, Boise, ID; and K. E. SAXTON, Research Hydrologist, USDA-Agricultural Research Service, Pullman, WA.

The impacts of tillage, residue, solutes, topographic and atmospheric conditions on soil freezing can be fully understood only with a detailed physical process model of a snow-residue-soil system. Such a model may currently be too complex and require data not readily available for many routine applications. However, detailed physically-based models form the basis for improved, simplified models after the complex interactions of energy and moisture transfer are understood and the effects of simplifying assumptions and approximations are evaluated. Additionally, it is often possible to interpolate commonly available climatic data and site descriptions to form useable data sets for the more comlex models.

A one-dimensional Simultaneous Heat And Water (SHAW) model is presented which simulates the interrelated heat, water and solute transfer through snow, residue and soil. The SHAW model was developed to: predict the effects of tillage and residue on soil freezing; obtain a better understanding of the interrelated processes of heat, water and solute transfer within a snow-residue-soil system; and evaluate management options for soil and water conservation. The model predicts hourly soil freezing depths, snow depths, evaporation and profiles of temperature, water, ice and solutes. Model validation is discussed in a companion paper (Part II) by Flerchinger and Saxton (1989).

## MODEL DEVELOPMENT

The physical system described by the SHAW model consists of a vertical, one-dimensional profile extending from the snow, residue or soil surface to a specified depth within the soil. The system is represented by integrating detailed physics of snow, residue and soil into one simultaneous solution. The interrelated heat, water and solute fluxes are computed throughout the system.

The model is sufficiently flexible to represent a broad spectrum of farmland conditions from a homogeneous bare soil to a highly layered tilled soil covered with residue and overlain by a snowpack. Transpiring plant canopies are not currently included. The system, illustrated in Fig. 1, may or may not have snow, crop residue or a tillage layer.

Heat and water flux to the system are determined from atmospheric conditions above the upper boundary and soil conditions at the lower boundary. A layered system is established through the snow, residue and soil and each layer is represented by an individual node. Liquid water, water vapor, heat and solute flux between layers are computed in hourly time steps and balanced with changing conditions within the layers. Flux equations written in implicit finite-difference form are solved

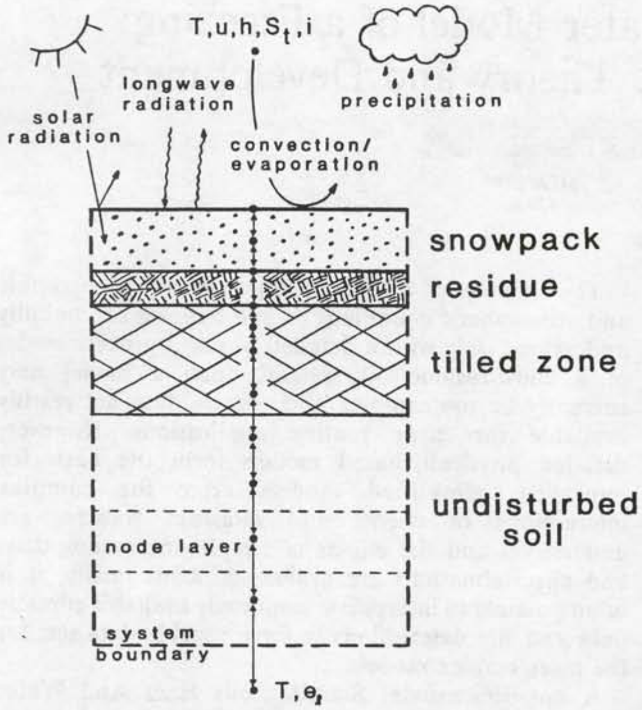


**Fig. 1—Physical description of the system described by the SHAW model.**

iteratively. The following sections discuss boundary fluxes and heat, water and solute transfer through the system.

**Boundary Fluxes**

Hourly values of air temperature, wind speed, relative humidity, solar radiation and precipitation define the upper boundary condition of the system. Absorbed solar radiation, net long-wave radiation and convective heat and moisture transfer at the surface are determined from these hourly measurements. Interception and infiltration of precipitation are computed at the end of the time step.

Solar radiation is often the major source of heat affecting the dynamics of a snow-residue-soil system. Solar radiation absorbed by the surface depends on the amount of incident direct and diffuse radiation, and the albedo, slope and aspect of the surface. Measured total radiation is separated into direct and difuse components using a method presented by Bristow et al. (1985).

Albedo of snow may be calculated from snow density (Anderson, 1976). Shallow snowpacks often do not completely cover a rough surface. For shallow snow depths, the albedo of the surface is adjusted for the exposed underlying material (residue or soil) based on the fraction of surface covered by snow:

$$F_{sp} = (d_{sp}/d_{min})^a \quad \ldots\ldots [1]$$

where $d_{sp}$ is the depth of snow, $d_{min}$ is the minimum depth of snow required for 100% cover and a is an empirical coefficient. Penetration and absorption of net solar radiation at the snow surface decreases exponentially with depth.

Penetration of direct and diffuse radiation in crop residues depends on the albedo of the residue elements and the fraction of surface area covered by residues. Radiation reflected and scattered by each layer may be absorbed by adjacent residue layers or lost to the atmosphere. A solar radiation balance is computed by considering the direct, and upward and downward diffuse radiation fluxes above and below each residue layer, as described by Bristow et al. (1986).

Net long-wave radiation exchange between the atmosphere and ground surface can be a significant component of the energy budget, particularly during nocturnal cooling. Long-wave radiation emitted by the atmosphere is estimated from the Stefan-Boltzman law with clear-sky atmospheric emissivity based on air temperature (Idso and Jackson, 1969). Since clouds have an emissivity very close to unity, the emissivity of cloudy skies is somewhere between that for clear sky and unity depending on the fraction of cloud cover. Cloud cover is estimated as a daily average based on a ratio of observed over potential solar radiation.

Net long-wave radiative flux at the snow or soil surface depends on radiation flux from the atmosphere, emissivity of the material and surface temperature. Transmission and adsorption of long-wave radiation in residue is similar to solar radiation, with the exception that scattering of long-wave radiation can be ignored and long-wave emittance must be considered. A long-wave radiation balance is calculated for each residue layer based on the fluxes incident on and emitted by each side of the nodal layer.

Convective heat and vapor transfer between the snow, residue or soil surface and the atmosphere are affected by atmospheric turbulence and eddy exchange. Heat flux, H ($W/m^2$), is calculated using the expression (Campbell, 1977)

$$H = \rho_a c_a \frac{(T - T_a)}{r_H} \quad \ldots\ldots [2]$$

where

$\rho_a$, $c_a$, $T_a$ = density ($kg/m^3$), specific heat (J/kg/C), and temperature (°C) of the air
T = surface temperature
$r_H$ = resistance to heat transfer (s/m).

Vapor flux, E ($kg/m^2$) from the surface is calculated by

$$E = \frac{\rho_{vs} - \rho_{va}}{r_v} \quad \ldots\ldots [3]$$

where

$\rho_{vs}$ and $\rho_{va}$ = surface and atmospheric vapor density ($kg/m^3$)
$r_v$ = resistance to vapor transfer (s/m).

The resistances $r_v$ and $r_H$ are assumed equal and depend on atmospheric stability, which is the ratio of thermally induced to mechanically induced turbulence (Businger, 1975; and Campbell, 1977).

**Heat Flux Within the System**

Net heat flux into each layer of snow, residue or soil must be balanced with heat stored as a result of temperature increase and/or phase change within each layer. Heat flux equations are solved simultaneously using an iterative Newton-Raphson technique. Discussion of the heat flux equation for snow, residue and soil layers follows.

*Snowpack*

The heat flux equation for a small layer within a snowpack is

$$\frac{\partial}{\partial z}\left(k_{sp}\frac{\partial T}{\partial z}\right) + S = \rho_{sp} c_i \frac{\partial T}{\partial t} + \rho_\ell L_f \frac{\partial w_{sp}}{\partial t}$$

$$+ L_s \left(\frac{\partial \rho_v}{\partial t} + \frac{\partial q_v}{\partial z}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots [4]$$

where the terms ($W/m^3$) represent, respectively: net thermal conduction into a layer; a source/sink term (which may include solar and long-wave radiation); specific heat stored due to a temperature increase; latent heat required to melt snow; and net latent heat of sublimation. (Variables are defined in Table 1.) Development of equation [4] assumes a constant snow depth over the time step and neglects heat transferred by water movement. At the end of each hour within the SHAW model, the thickness of each layer is adusted for melt, and temperature and water content are adjusted for water percolating through the snowpack. Initial conditions necessary for the solution of equation [4] include the temperature, water content and density of each layer. Boundary conditions for the snowpack include heat and vapor flux calculated between the atmosphere and the snow surface and temperature of the residue or soil beneath the snow.

The primary mechanisms for energy transfer within a snowpack is thermal conduction between and within ice crystals. Thermal conductivity of snow has been empirically related to density by many researchers, although geometry of the snow crystals is important as well. An expression of the form

$$k_{sp} = a_{sp} + b_{sp}(\rho_{sp}/\rho_\ell)^{c_{sp}} \quad \ldots\ldots\ldots\ldots\ldots\ldots [5]$$

is suggested by Anderson (1976) and will fit many empirically derived correlations. Here, $\rho_{sp}$ and $\rho_\ell$ are the density of snow and water, respectively, and $a_{sp}$, $b_{sp}$, and $c_{sp}$ are empirical coefficients with suggested values of 0.021 W/m/C, 2.51 W/m/C, and 2.0, respectively.

Latent heat transfer by sublimation results from vapor transfer through the snowpack in response to vapor density gradients caused by temperature gradients. Vapor density in snow is assumed equal to the saturated vapor density over ice, and therefore is a function of only temperature. Warmer parts of the snowpack have a higher vapor density, causing vapor diffusion toward cooler parts where, due to over-saturation, sublimation will occur and latent heat is released.

*Residue*

The one-dimensional heat flux equation for an infinitely small residue layer is expressed as

$$\frac{\partial}{\partial z}\left(k_r\frac{\partial T}{\partial z}\right) + S = C_r \frac{\partial T}{\partial z} + L_v E_r \quad \ldots\ldots\ldots\ldots [6]$$

where the terms ($W/m^3$) represent respectively: net heat transferred into a layer by conduction and convection; a source/sink term (which may include solar and long-wave radiation); energy stored by specific heat of residue; and latent heat of evaporation from the residue elements, i.e. straw. (Variables are defined in Table 1.) Equation [6] includes the assumption that residue elements and surrounding air voids within a layer are in thermal equilibrium. Boundary conditions for the residue are heat flux from the atmosphere or snow above the residue and soil temperature below the residue.

Heat is transferred through the residue by conduction through residue elements and convection through air voids. The relative magnitude of these two processes depends on wind speed within the residue, and density and moisture content of the residue. Based on results from Kimball and Lemon (1971), Bristow et al. (1986) assumed that thermal convection through crop residue increased linearly with wind speed, but neglected the effect of residue density. The equation modified for this model for density of the residue is

$$k_v = k_a(1 + 0.007T)(1 + 4u_r)(1 - \rho_r/\rho_{rs}) \quad \ldots\ldots\ldots [7]$$

where

- $k_v$ = convective heat transfer coefficient (W/m/C
- $k_a$ = thermal conductivity of still air (W/m/C)
- $u_r$ = wind speed within the residue (m/s)
- $\rho_r$ = bulk density of the residue ($kg/m^3$)
- $\rho_{rs}$ = specific density of residue (nominally 170 $kg/m^3$ as given by Unger and Parker, 1976).

Wind speed at the surface of the residue is calculated from atmospheric wind speed measurements using the traditional logarithmic profile equation (Campbell, 1977), and is assumed to decrease linearly within the residue to a value of zero at the soil surface.

Thermal conduction within the residue is dependent largely on residue density and moisture content and is calculated as a weighted average of the conductivities of residue and water:

$$k_t = k_{rs}(\rho_r/\rho_{rs}) + k_\ell\, w(\rho_r/\rho_\ell) \quad \ldots\ldots\ldots\ldots\ldots [8]$$

where

- $k_t$ = thermal conductivity of the residue layer (W/m/C)
- $k_{rs}$ = thermal conductivity of residue at specific density (W/m/C)
- $k_\ell$ = thermal conductivity of water (W/m/C)
- w = moisture content of the residue (kg/kg).

The total heat transfer coefficient of the residue, $k_r$ (equation [6]) is the sum of the convection and conduction coefficients.

Latent heat is required to evaporate liquid water from the residue elements to air voids within the residue layer. The rate of evaporation depends on the vapor density within the void spaces and the moisture content of the residue and will be presented in the water balance section.

*Soil*

The general heat flux equation for a layer of potentially freezing soil is written as:

$$\frac{\partial}{\partial z}\left(k_s\frac{\partial T}{\partial z}\right) + \rho_\ell c_\ell \frac{\partial (q_\ell T)}{\partial z} + S = C_s \frac{\partial T}{\partial t}$$

$$- \rho_i L_f \frac{\partial \theta_i}{\partial t} + L_v \left(\frac{\partial \rho_v}{\partial t} + \frac{\partial q_v}{\partial z}\right) \quad \ldots\ldots\ldots\ldots [9]$$

where the terms ($W/m^3$) represent, respectively: net thermal conduction into a layer; net thermal advection into a layer due to water flux; a source/sink term (which may include solar and long-wave radiation); specific heat stored due to a temperature increase; latent heat required to freeze water; and latent heat of evaporation in layer. (Variables are defined in Table 1). Boundary conditions for the soil are heat and vapor flux from the atmosphere, snow or residue above the soil and a specified temperature at the lower boundary.

Thermal conductivity of the soil is calculated using the theory presented by DeVries (1963). A moist soil (water content greater than approximately 0.05 $m^3/m^3$ for sand and 0.15 $m^3/m^3$ for clay) can be conceptualized as a continuous medium of liquid water with granules of soil, crystals of ice and pockets of air dispersed throughout. The thermal conductivity of such an idealized model is expressed as:

$$k_s = \frac{\Sigma m_j k_j \theta_j}{\Sigma m_j \theta_j} \quad \ldots\ldots\ldots\ldots\ldots\ldots [10]$$

where $m_j$, $k_j$ and $v_j$ are the weighting factor, thermal conductivity, and volumetric fraction of the $j^{th}$ soil constituent, i.e., soil minerals, water, ice and air. Values for $m_j$ may be calibrated from thermal conductivity data or estimated from DeVries (1963). Thermal conductivity for soils with lower water contents is interpolated between moist and oven-dry conditions. Volumetric heat capacity, $C_s$, of soil is the sum of the volumetric heat capacities of the soil constituents:

$$C_s = \Sigma \rho_j c_j \theta_j \quad \ldots\ldots\ldots\ldots\ldots\ldots [11]$$

where $\rho_j$, $c_j$ and $v_j$ are the density ($kg/m^3$), specific heat capacity (J/kg/C) and volumetric fraction ($m^3/m^3$) of the $j^{th}$ soil constituent.

Due to matric and osmotic potentials, soil water exists in equilibrium with ice at temperatures below the normal freezing point of bulk water and over the entire range of freezing temperatures normally encountered. A relation between liquid content and temperature must therefore be defined before the latent heat term can be determined. When ice is present, soil water potential remains in equilibrium with the vapor pressure over pure ice and is given by the freezing point depression equation (Fuchs et al., 1978):

$$\phi = \pi + \psi = \frac{L_f T}{g(T+273.16)} \quad \ldots\ldots\ldots\ldots [12]$$

where

$\phi$ = total soil water potential (m)
$\pi$ = soil water osmotic potential (m)
$\psi$ = soil matric potential (m)
$L_f$ = latent heat of fusion (J/kg)
g = acceleration of gravity ($m/s^2$)
T = soil temperature in degrees Celsius (degrees from the freezing point of pure bulk water).

Given osmotic potential calculated from equation [20], soil temperature defines the matric potential and, therefore, liquid water content. If total water content is known, ice content and the latent heat term can be determined.

**Water Flux Within the System**

Net water flux into each residue and soil layer is balanced with an increase in total water stored in the layer. Water balance of the snowpack is computed at the end of each hour. The following sections describe the water flux equations for the snowpack, residue and soil layers.

*Snowpack*

For each layer within the snowpack, density and ice content are assumed constant during each h time step while change in liquid content due to melt is computed from the energy balance. At the end of each hour, the thickness and density for each layer are adjusted for vapor transfer and change in liquid water content. Excess liquid water is routed through the snowpack using attenuation and lag coefficients to determine snowcover outflow, and density of the snow is then adjusted for compaction and settling (Anderson, 1976).

*Residue*

Liquid transport through residue is assumed negligible (with the exception of transmission of rainfall and snowmelt through residue). The flux equation for water vapor in the air voids of a residue layer is expressed as

$$\frac{\partial}{\partial z}\left(K_v \frac{\partial \rho_v}{\partial z}\right) + E_r = \frac{\partial \rho_v}{\partial t} \quad \ldots\ldots\ldots\ldots [13]$$

where the terms ($kg/m^3/s$) represent, respectively: net vapor flux into a residue layer; evaporation rate from residue elements; and rate of change in vapor density. (Variables are defined in Table 1). Boundary conditions for water flux through the residue are vapor flux from the atmosphere or snow above the residue and vapor density within the soil below the residue.

Convective transfer of vapor through residue is analogous to convective heat transfer (equation [7]). The convective vapor diffusion coefficient is related to the convective heat transfer coefficient by

$$K_v = k_v / \rho_a c_a \quad \ldots\ldots\ldots\ldots\ldots\ldots [14]$$

where

$k_v$ = convection heat transfer coefficient through residue
$\rho_a$ and $c_a$ = density and specific heat of air.

Evaporation rate from a residue layer of thickness $\Delta z$ is expressed as (Bristow et al., 1986)

$$E_r \Delta z = K_r (h\rho_v' - \rho_v) \quad \ldots\ldots\ldots\ldots [15]$$

where

$K_r$ = vapor conductance between residue elements and the air voids (m/s)
h = relative humidity of air in equilibrium with the residue elements
$\rho_v'$ = saturated vapor density ($kg/m^3$) at the temperature of the residue
$\rho_v$ = vapor density ($kg/m^3$) of the air voids in the residue.

Relative humidity of the residue is determined from water content, and is based on water potential of the residue. Myrold et al. (1981) presented empirical equations for the water potential of wheat straw based on water content.

*Soil*

The equation for liquid and vapor flow through a frozen or unfrozen, unsaturated, heterogeneous, vertical soil profile is

$$\frac{\partial}{\partial z}\left[K\left(\frac{\partial \psi}{\partial z}+1\right)\right]+\frac{1}{\rho_\ell}\frac{\partial q_v}{\partial z}+U = \frac{\partial \theta_\ell}{\partial t}+\frac{\rho_i}{\rho_\ell}\frac{\partial \theta_i}{\partial t} \quad \ldots\ldots [16]$$

where the terms ($m^3/m^3/s$) represent, respectively: net liquid flux into a layer; net vapor flux into a layer; a source/sink term (which may include root uptake with future model development); rate of change of volumetric liquid content; and rate of change of volumetric ice content. (Variables are defined in Table 1). The above equation includes assumptions that liquid transport in response to thermal and osmotic gradients are negligible. However, thermal gradients in frozen soil will indirectly cause liquid water migration by creating water potential gradients. Boundary conditions for water flux through the soil are vapor flux from the atmosphere, snow or residue above the soil and a specified soil water content at the lower boundary.

Water flux is computed from a matric potential gradient times the hydraulic conductivity. Matric potential is calculated from water content using (Brooks and Corey, 1966)

$$\psi = \psi_e\left(\frac{\theta_\ell}{\theta_s}\right)^{-b} \quad \ldots\ldots [17]$$

where

$\psi_e$ = air entry potential (m)
$\theta_\ell$ = liquid water content ($m^3/m^3$)
$\theta_s$ = saturated water content ($m^3/m^3$)
b = a pore-size distribution index.

Unsaturated conductivity, K, is determined from matric potential using the following expression:

$$K = K_s\left(\frac{\psi_e}{\psi}\right)^{n} = K_s\left(\frac{\theta_\ell}{\theta_s}\right)^{bn} \quad \ldots\ldots [18]$$

where

$K_s$ = saturated hydraulic conductivity (m/s)
n = 2 + 3/b (Campbell, 1974).

Water flow in frozen soil is assumed analogous to that in unsaturated, unfrozen soil (Harlan, 1973; Cary and Mayland, 1972; and Miller, 1963), and the relations for matric potential and hydraulic conductivity of unfrozen soils are assumed valid for frozen soil. However, based on results of Bloomsburg and Wang (1969), hydraulic conductivity is set to zero if $(\theta_s - \theta_i) < 0.13$, where $\theta_i$ is ice content ($m^3/m^3$).

Vapor flux through soil, $q_v$, may be expressed as flux due to a potential gradient plus flux due to a temperature gradient:

$$q_v = q_{vp} + q_{vT} = -D_v\rho_v'\frac{dh}{dz} - \zeta D_v h s\frac{dT}{dz} \quad \ldots\ldots [19]$$

where

$q_{vp}$ and $q_{vT}$ = vapor flux due to a water potential gradient and a temperature gradient respectively ($kg/m^2/s$)
$D_v$ = vapor diffusivity in the soil ($m^2/s$)
$\rho_v'$ = saturated vapor density ($kg/m^3$) at soil temperature T
h = relative humidity within the soil
s = change in saturated vapor density with respect to temperature ($d\rho_v'/dT$ in $kg/m^3/K$)
$\zeta$ = enhancement factor for vapor flux in response to a temperature gradient (Cass et al., 1984).

Unknowns in the soil water flux equation [16] are the change in liquid and ice contents over the time step. With two unknowns, additional information is needed for a solution. When ice is present, total water potential is a function of temperature. The osmotic potential of the soil solution may be expressed as

$$\pi = -cRT_K/g \quad \ldots\ldots [20]$$

where

c = solute concentration (moles/kg) in the soil solution
R = universal gas constant (8.3143 J/mole/K)
$T_K$ = temperature in degrees Kelvin.

Combining equations [12], [17], and [20], $v_\ell$ may be expressed as a function of temperature:

$$\theta_\ell = \theta_s\left[\frac{\dfrac{L_f T}{T+273.16}+cRT_K}{g\psi_e}\right]^{-1/b} \quad \ldots\ldots [21]$$

This equation defines the maximum liquid water content for sub-zero temperatures, thus any additional water is ice.

**Solute Flux Within the System**

The SHAW model includes an accounting for solutes with or without adsorption by the soil matrix. Three processes of solute transfer computed by the model are molecular diffusion, convection and hydrodynamic dispersion. Solution of the solute flux equation was patterned after Bresler (1973). Several species of solutes may be simulated simultaneously, however solutes are assumed non-interacting with each other or soil organisms.

**Numerical Solution**

The heat, water and solute flux equations for each layer in the snow-residue-soil system are written in implicit finite difference form (Flerchinger, 1987) and solved using an iterative Newton-Raphson technique. A balance equation for each layer is written in terms of unknown end-of-time-step values within the layer and its neighboring layers. Partial derivatives of the flux equations with respect to the unknown end-of-time-step values are computed, forming a tri-diagonal matrix from which the Newton-Raphson approximations for the unknown values are computed. Iterations continue until successive approximations for each layer are within a prescribed tolerance.

The SHAW model uses a maximum time step of one hour, and smaller time steps may be specified. Time steps within an hour are halved if convergence is not met before ten iterations of a time step. The computational procedure for each hour time step may be summarized as follows:

1. Compute absorbed solar radiation.
2. Compute net long-wave radiation exchange.
3. Compute turbulent transfer of heat and water surface.
4. Set up matrix for partial derivatives of heat flux equations and calculate approximation to end-of-time-step temperatures (and liquid water content for melting snow).
5. Set up matrix for partial derivatives of water flux equations and calculate end-of-time-step values for vapor density in the residue, matric potential in unfrozen soil, and ice content in frozen soil (matric potential and liquid content in frozen soil are defined by temperature, and the change in ice content over the time step is used in the soil heat flux equation in the subsequent iteration).
6. If sucessive approximations do not meet prescribed tolerance, return to step 2.
7. Set up and solve matrix for solute flux equations for each type of solute.
8. If solute concentrations changed significantly, return to step 2.
9. Add any new snow to snowpack; adjust snow for melt, vapor transfer and compaction; adjust residue water content for rainfall or snowmelt interception; and adjust soil water, temperature and solutes for infiltration.

A more detailed description of the solution procedure, including all partial derivatives, is given by Flerchinger (1987). A complete solution requires establishing a parameter set to describe the site conditions, a data set of hourly weather conditions and initial profiles of soil moisture, temperature and solute concentration. A listing of the model, written in FORTRAN 77, may be obtained by contacting the authors.

## SUMMARY

A detailed Simultaneous Heat And Water (SHAW) model was developed to simulate the complex interactions of energy, moisture and solutes in a snow-residue-soil system. The model was developed specifically for cold-weather, soil-freezing applications, but the theory applies equally to warm weather and total year simulations. Hourly soil frost depths, snow depths, evaporation and profiles of temperature, water, ice and solutes are predicted using hourly weather data by computing the heat, water and solute transfer through snow, residue and soil. A wide range of agricultural field situations can be represented using readily definable site parameters. The model's capability for predicting soil freezing for widely varying tillage and residue treatments is discussed by Flerchinger and Saxton (1989) in a companion paper (Part II).

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TABLE 1. List of Symbols

| | |
|---|---|
| $a$ | empirical coefficient |
| $b$ | pore-size distribution index |
| $a_{sp}$ | empirical coefficient (0.21 W/m/C) |
| $b_{sp}$ | empirical coefficient (2.51 W/m/C) |
| $c$ | solute concentration of soil solution (moles/kg) |
| $c_\ell$ | specific heat capacity of water (4200 J/kg/C) |
| $c_a$ | specific heat of air (J/kg/C) |
| $c_i$ | specific heat of ice (J/kg/C) |
| $c_j$ | specific heat capacity of $j^{th}$ soil constituent (J/kg/C) |
| $c_{sp}$ | empirical coefficient (2.0) |
| $C_r$ | volumetric heat capacity of residue ($J/m^3/C$) |
| $C_s$ | volumetric heat capacity of soil ($W/m^3/C$) |
| $d_{min}$ | minimum depth of snow required for 100% cover (m) |
| $d_{sp}$ | depth of snow (m) |
| $D_v$ | vapor diffusivity in the soil ($m^2/s$) |
| $E$ | vapor flux from the surface ($kg/m^2$) |
| $E_r$ | rate of evaporation from residue elements (i.e., straw and chaff) to the void spaces within the residue ($kg/m^3/s$) |
| $F_{sp}$ | fraction of surface covered by snow |
| $g$ | acceleration of gravity ($m/s^2$) |
| $h$ | relative humidity |
| $H$ | convective heat flux from surface ($W/m^2$) |
| $k_a$ | thermal conductivity of still air (W/m/C) |
| $k_j$ | thermal conductivity of $j^{th}$ soil constituent (W/m/C) |
| $k_\ell$ | thermal conductivity of water (W/m/C) |
| $k_r$ | combined thermal conduction/convection term for residue (W/m/C) |
| $k_{rs}$ | thermal conductivity of residue at specific density (W/m/C) |
| $k_s$ | thermal conductivity of soil (W/m/C) |
| $k_{sp}$ | thermal conductivity of snow (W/m/C) |
| $k_t$ | thermal conductivity of residue layer (W/m/C) |
| $k_v$ | convective heat transfer coefficient (W/m/C) |
| $K$ | unsaturated hydraulic conductivity (m/s) |
| $K_r$ | vapor conductance between residue elements and air voids (m/s) |
| $K_s$ | saturated hydraulic conductivity (m/s) |
| $K_v$ | convective vapor diffusion coefficient ($m^2/s$) |
| $L_f$ | latent heat of fusion (335,000 J/kg) |
| $L_s$ | latent heat of sublimation (2,835,000 J/kg) |
| $L_v$ | latent heat of vaporization (2,500,000 J/kg) |
| $m_j$ | weighting factor of the $j^{th}$ soil constituent |
| $n$ | $2 + 3/b$ (Campbell, 1974) |
| $q_\ell$ | downward liquid water flux (m/s) |
| $q_v$ | downward water vapor flux ($kg/m^2/s$) |
| $q_{vp}$ | vapor flux due to water potential gradient ($kg/m^2/s$) |
| $q_{vT}$ | vapor flux due to temperature gradient ($kg/m^2/s$) |
| $r_H$ | resistance to convective heat transfer from surface (s/m) |
| $r_v$ | resistance to vapor transfer from surface (s/m) |
| $R$ | universal gas constant (8.3143 J/mole/K) |
| $s$ | change in saturated vapor density with respect to temperature ($d\rho_v'/dT$ in $kg/m^3/K$) |
| $S$ | source/sink term in heat flux equation ($W/m^3$) |
| $t$ | time (sec) |
| $T$ | temperature (°C) |
| $T_a$ | atmospheric temperature (°C) |
| $T_k$ | temperature (K) |
| $u_r$ | wind speed within residue (m/s) |
| $U$ | source/sink term in water flux equation ($m^3/m^3/s$) |
| $w$ | moisture content of residue (kg/kg) |
| $w_{sp}$ | volumetric liquid fraction in the snow (m/m) |
| $z$ | depth from surface (m) |
| $\zeta$ | enhancement factor for temperature gradient vapor flux |
| $\theta_i$ | volumetric ice content (m/m) |
| $\theta_j$ | volumetric fraction of $j^{th}$ soil constituent ($m^3/m^3$) |
| $\theta_\ell$ | volumetric liquid water content ($m^3/m^3$) |
| $\theta_s$ | saturated water content ($m^3/m^3$) |
| $\pi$ | soil water osmotic potential (m) |
| $\rho_a$ | density of air ($kg/m^3$) |
| $\rho_\ell$ | density of liquid water (1000 $kg/m^3$) |
| $\rho_i$ | density of ice (920 $kg/m^3$) |
| $\rho_j$ | density of $j^{th}$ soil constituent ($kg/m^3$) |
| $\rho_r$ | bulk density of the residue ($kg/m^3$) |
| $\rho_{rs}$ | specific density of residue (170 $kg/m^3$) |
| $\rho_{sp}$ | density of ice fraction of snowpack ($kg/m^3$) |
| $\rho_v$ | vapor density within the snow, residue or soil ($kg/m^3$) |
| $\rho_v'$ | saturated vapor density ($kg/m^3$) |
| $\rho_{va}$ | atmospheric vapor density ($kg/m^3$) |
| $\rho_{vs}$ | surface vapor density ($kg/m^3$) |
| $\phi$ | total soil water potential (m) |
| $\psi$ | soil matric potential (m) |
| $\psi_e$ | air entry potential (m) |